LONG TERM TRENDS IN EQUITY PRICES

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This model is inspired by the historic tendency of the ratio of yields in gilt and equity investments to average 2.Deviations outside the 1.5 to 2.5 range in the U.K. have always presaged sharp correction.The model focusses on yields,not earnings incorporating retained earnings into future yield expectations and seeks to establish rational price levels in equity markets by comparing their returns over time to those available in other investments,particularly gilts.

The model assumes that all investors have expectations of future interest rates and dividends of which they are certain (or at least risk of deviations from these expectations is symmetric on either side),that they are rational,and that after some point in time in the future,perhaps the very distant future,expected interest rates and dividend growth rates become constant over time (e.g. our expectations of these variables in 2200 are the same as those in 2250).ß and r represent yield dividends and interest rates respectively which are functions of time(t).By the last assumption there exists some t' such that if t>t' r=r' ,$\beta=\beta'(1+k)^t$,where ß,k and r' are constants.

The model requires expected equity returns to be equal to expected gilt returns over all time periods or turning this on its head it requires the price of an equity at a point in time to equal to its price and cumulative return at any other point in time discounted at the interest rates implicit in gilt prices.the valuation at t=0,P_0 is given below.t=(1,2,..........)

$$P0=\int_0^1 \frac{\beta dt}{(1+r)^t} + \frac{\Delta P+P0}{\int_0^1(1+r)^t dt} = \int_0^1 \frac{\beta dt}{(1+r)} + \frac{P1}{\int_0^1(1+r)^t dt} \quad [\Delta P=P1-P0]$$

$$\frac{P1}{\int_0^1(1+r)^t dt} = \int_1^2 \frac{\beta}{(1+r)^t} dt + \left[P2-\frac{P1}{\int_0^1(1+r)^t dt} + \frac{P1}{\int_0^1(1+r)^t dt}\right] \frac{1}{\int_1^2(1+r)^t dt}$$

$$= \int_1^2 \frac{\beta}{(1+r)^t} dt + \frac{P2}{\int_1^2(1+r)^t dt}$$

similarly, $$\frac{Pi}{\int_{i-1}^{i}(1+r)^t dt} = \int_i^{i+1} \frac{\beta}{(1+r)^t} dt + \frac{Pi+1}{\int_i^{i+1}(1+r)^t dt}$$

Therefore $$P0 = \int_0^1 \frac{\beta}{(1+r)^t} dt + \left(\int_1^2 \frac{\beta}{(1+r)} dt + \left(\int_1^3 \frac{\beta}{(1+r)^t} dt \ldots\ldots\ldots\ldots\right.\right.$$

i.e. $$P0 = \int_0^\infty \frac{\beta\, dt}{(1+r)^t} = \int_0^{t'} \frac{\beta\, dt}{(1+r)^t} + \int_{t'}^{\infty} \frac{\beta'(1+k)^t}{(1+r')^t} dt \quad \text{..Deduction 1}$$

But P0 is infinite if $\lim_{t \to \infty} \dfrac{\beta'(1+k)^t}{(1+r)^t} \neq 0$

P0 is finite,therefore $(1+k)^t < (1+r')^t$ for all t, t >t'
therefore k<r' ...... Deduction 2

If t+t' and dividends are paid only where t=1,2,.............(i.e. is an integer) then

$$P0 = \sum_{t=0}^{\infty} \frac{\beta'(1+k)^t}{(1+r)^t} = \sum_{t=0}^{\infty} \frac{\beta'(1+k)^t}{(1+r)^t}$$

Therefore $P0 = \left[\dfrac{1}{1 - \left(\dfrac{1+k}{1+r'}\right)}\right]\beta'$ (Sum of geometric progression to infinity)

$$= \frac{\beta'(1+r)}{(1+r)-(1+k)}$$

Therefore $P0 = \dfrac{\beta'(1+r)}{r'-k}$ ....Deduction 3

Deduction 1 essentially means that a rational valuation of an equity in this model involves looking only at its future yield because these will cause price fluctuations.Deduction 2 is more interesting;it means it is irrational to expect dividend growth rates to exceed interest rates for ever.This is a very significant conclusion as it is almost certainly defied in respect of long dated gilts in countries with low interest rates e.g. West Germany or Japan.Deduction 3 applies to a special case even within the model i.e. where expectations of interest rates and dividend growth rates are that they are constant over time.It would indicate however that where r' and k are close extreme sensitivity of price to changes if r' and k is rational.
Although this model may partly account for long term equity trends it does not adequately explain volatility in contemporary equity markets ,because these add on risk premia.These are not primarily because of uncertainty in predicting the r and ß functions but because of feared irrationality in the markets.Risk premia imply further irrationality so irrationality and rationality are both self fulfilling prophesies.Hence we can develop the above model to include features of cobweb models in it;movements towards or away from the model itself are self perpetuating and lead ultimately to stability or wild volatilty.